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Adaptive Stable PID Controller of a class of SISO nonaffine nonlinear systems

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Abstract **- This paper presents a stable self-tuning PID control scheme for a class of uncertain continuous-time single-input single-output (SISO) nonaffine nonlinear dynamic systems. Based on the implicit function theory, the existence of an ideal controller, that can achieve control objectives, is firstly shown. Since the implicit function theory guarantees only the existence of the ideal controller and does not provide a way for constructing it, a PID controller is employed to approximate this unknown ideal control law. The three PID control gains are the adjustable parameters and they are updated online with a stable adaptation mechanism designed to minimize the error between the unknown ideal controller and the used PID controller. The effectiveness of the proposed adaptive control scheme is demonstrated through the simulation of a simple nonaffine nonlinear system.**

Index Terms **- PID control, adaptation mechanism, Nonaffine nonlinear systems.**

I. INTRODUCTION

The PID control algorithm is the well known approach in the automatic control field. Since 1940, emerge of process control, PID controllers are used in most of the feedback loops of process industries despite continual advances in control theory. These controllers are preferred because of their versatility, simple structure, high reliability and easy implementation on analog or digital platforms. Nowadays, around the 90% of industrial objects are controlled by PID controllers [1]. The key idea of designing a PID controller is the choice of three parameters, i.e. proportional gain K_p, integral gain K_I, and derivative gain K_d. To yield satisfactory control results, the values of K_p, K_I and K_d must be tuned. Over the past half century, researchers have sought the key techniques for PID tuning. These methods can be classified as: (1) empirical methods such as the Ziegler–Nichols method [2][3] and the internal model control [3], (2) analytical methods such as root locus based techniques [3], (3) methods based on optimization such as the iterative feedback tuning [4][5], genetic algorithm tuning technique [6][7], chaotic optimization [8] and optimization with the extended Kalman filter (EKF) [9], (4) self-tuning methods [10][11][12]. However, the empirical techniques, the analytical methods and the methods based on optimization have some particular conditions regarding the plant models, such as dead time or transport lag, fast and slow poles, real and complex conjugated zeros and poles, as well as unstable poles, etc. These conditions make the previous methods non-general and only suitable in the case of linear systems with a known model.

By reason of the progress in the industrial applications, there are many processes with time-variant or nonlinear characteristics and, hence, the PID controller tuned with conventional tuning methods becomes inefficient for these systems. In order to solve this problem, the adaptive PID controller design has received wide attention. The common design idea of adaptive PID controller is to adjust PID parameters according to varying system states to obtain better control effects. For SISO systems, there are many kinds of adaptive PID control methods that have been proposed [11][13][14][15]. In the paper [11], Chang et al. developed a self-tuning PID control for a class of continuous-time SISO nonlinear systems based on the Lyapunov approach. The stability of the closed-loop system is analyzed and guaranteed by introducing a supervisory control and a modified adaptation law with projection. In [13], Chang and Yan proposed an adaptive robust PID controller design based on a sliding mode for a class of uncertain chaotic SISO nonlinear systems. In [14], Mizumoto et al. proposed a design scheme of an adaptive PID control system for discrete-time SISO nonlinear systems based on the output feedback strictly passive property of the controlled system. In [15], Mizumoto et al. presented the design of an adaptive PID control system with a parallel feedforward compensator for discrete time SISO systems and its application to water level control of a 3-tank system.

In this paper, we introduce a stable self-tuning PID control scheme for a class of uncertain SISO nonaffine nonlinear dynamic systems. The basic idea is to use PID controllers to approximate an unknown ideal controller. At first, the implicit function theory has been used to prove the existence of an ideal controller that can achieve control objectives. Within this scheme, the adaptive laws of the gains K_p, K_I and K_d are designed, based on the gradient descent method, to directly minimizing the error between the unknown ideal controller and the used PID controllers. The overall closed-loop system stability is studied by using a Lyapunov approach. The proposed SISO self-tuning PID controller guarantees the boundedness of all variables in the closed-loop system and the convergence of the output tracking error to a small neighborhood of the origin.

This paper is organized as follows. Section II presents the control problem formulation and control objectives. The proposed self-tuning PID controller scheme is described in section III with its adaptive law and the stability analysis of the overall system. In section IV, the proposed control scheme is used to control a simple nonaffine nonlinear. Finally, section V concludes the paper.

I. PROBLEM STATEMENT

The class of uncertain nonaffine SISO nonlinear systems to be studied in this paper is represented in the following normal form:

$$\begin{cases} \dot{x}_1 = x_2, \dot{x}_2 = x_3, \ldots, \dot{x}_{n-1} = x_n \\ \dot{x}_n = f(\boldsymbol{x}, u) \\ y = x_1 \end{cases} \tag{1}$$

where $\boldsymbol{x} = [x_1, \ldots, x_n]^{\mathrm{T}} \in R^n$, is the state vector of the system in the normal form which is assumed available for measurement, $u \in R$ is the scalar control inputs, $y \in R$ is the scalar system output, and $f(\boldsymbol{x}, u)$ is an unknown smooth nonlinear function.

Differentiating y with respect to time for n times, until the inputs appear, one obtains the input-output form of (1) as

$$y^{(n)} = f(\boldsymbol{x}, u) \tag{2}$$

In this section, our goal is to design a control law $u(t)$ for system (1) such that the system output $y(t)$ follows a desired trajectory $y_d(t)$ while all signals in the closed-loop system remain bounded.

Throughout this paper we make the following assumptions regarding the system (1) and the desired trajectory $y_d(t)$.

Assumption 1. The function $f_u(\boldsymbol{x}, u) = \partial f(\boldsymbol{x}, u)/\partial u$ is nonzero and bounded for all $(\boldsymbol{x}, u) \in \Omega_x \times R$. This implies that $f_u(\boldsymbol{x}, u)$ is strictly either positive or negative for all $(\boldsymbol{x}, u) \in \Omega_x \times R$. Without loss of generality, it is assumed that the function $f_u(\boldsymbol{x}, u)$ is bounded as $0 < \delta_0 < f_u(\boldsymbol{x}, u) < \delta_1$ where δ_0 and δ_1 are some positive constants. Note that the result of this paper can be easily adapted to the case of systems with $-\delta_0 < f_u(\boldsymbol{x}, u) < -\delta_1 < 0$.

Assumption 2. The desired trajectory $y_d(t)$ and its time derivatives $y_d^{(i)}(t)$, $i = 1, \ldots, n$, are smooth and bounded.

Define the tracking error as

$$e(t) = y_d(t) - y(t) \tag{3}$$

and the filtered tracking error as

$$s(t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} e(t), \ \lambda > 0 \tag{4}$$

From (4), $s(t) = 0$ represents a linear differential equation whose solution implies that the tracking error $e(t)$ converges to zero with a time constant $(n-1)/\lambda$. In addition, the derivatives of $e(t)$ up to $n-1$ also converge to zero [16]. Thus, the control objective becomes the design of a controller to keep $s(t)$ at zero, therefore, the original stabilizing problem of the $n-$dimensional vector $\left[e \ldots e^{(n-1)}\right]^{\mathrm{T}}$, is reduced to that of keeping the scalar $s(t)$ at zero. Moreover, bounds on $s(t)$ can be directly translated into bounds on the tracking error. Specifically, if we have $|s(t)| \le \Phi$ where Φ is a positive constant, we can conclude that [16]: $\left|e^{(i)}(t)\right| \le 2^i \lambda^{(i-n+1)} \Phi$, $i = 0, \ldots, n-1$. These bounds can be reduced by increasing the design parameters λ.

The time derivative of the filtered error (4) can be rewritten as

$$\dot{s} = y_d^{(n)} + \beta_{n-1} e^{(n-1)} + \cdots + \beta_1 e^{(1)} - f(\boldsymbol{x}, u) \tag{5}$$

with

$$\beta_i = \frac{(n-1)!}{(n-i)!(i-1)!} \lambda^{n-i}, \ i = 1, \ldots, n-1.$$

Let a signal v be defined as

$$v = y_d^{(n)} + \beta_{n-1} e^{(n-1)} + \cdots + \beta_1 e^{(1)} + Ks + K_0 \tanh\left(s/\varepsilon_0\right) \quad (6)$$

where $K > 0$ and $K_0 > 0$, ε_0 is a small positive constant, and $\tanh(\cdot)$ is the hyperbolic tangent function.

Remark 1. The motivation of using the term $K_0 \tanh\left(s/\varepsilon_0\right)$ in the signal v , is to ensure some robustness, against modeling error, for the adaptive PID controller which will be proposed later. The term $K_0 \tanh\left(s/\varepsilon_0\right)$ is a smooth approximation of the discontinuous term $K_0 sign(s)$ usually used in robust control. So, K_0 is selected larger than the magnitude of the uncertainty and it will affect the convergence rate of the tracking error, and ε_0 is chosen very small to best approximate the sign function and it will affect the size of the residual set to which the tracking error will converge. The sign function $sign(\cdot)$ is not used here to avoid problems associated with it as chattering and solutions existence.

By adding and subtracting the term, $Ks + K_0 \tanh\left(s/\varepsilon_0\right)$, to the right-hand side of (5), we obtain

$$\dot{s} = -Ks - K_0 \tanh\left(s/\varepsilon_0\right) - \left(f\left(\boldsymbol{x},u\right) - v\right) \quad (7)$$

From Assumption 1 and the fact that the signal v , defined in (6), does not explicitly depend upon the control input u , the partial derivative of $f\left(\boldsymbol{x},u\right) - v$ with respect to the input u satisfies

$$\frac{\partial\left(f\left(\boldsymbol{x},u\right) - v\right)}{\partial u} = \frac{\partial f\left(\boldsymbol{x},u\right)}{\partial u} > 0 \quad (8)$$

Thus, based on the implicit function theorem [17,18], we know that the nonlinear algebraic equation $f\left(\boldsymbol{x},u\right) - v = 0$ is locally solvable for the input u for each $\left(\boldsymbol{x},u\right)$. Thus, there exists some ideal controller $u^*\left(\boldsymbol{x},v\right)$ satisfying the following equality for all $\left(\boldsymbol{x},u\right) \in \Omega_x \times R$:

$$f\left(\boldsymbol{x},u^*\left(\boldsymbol{x},v\right)\right) - v = 0 \quad (9)$$

Therefore, if the control input u is chosen as the ideal control law, i.e., $u = u^*$, the closed-loop error dynamic (7) is reduced to

$$\dot{s} = -Ks - K_0 \tanh\left(s/\varepsilon_0\right) \quad (10)$$

From which one can conclude that $s(t) \to 0$ as $t \to \infty$ and, therefore, $e(t)$ and all its derivatives up to $n-1$ converge to zero [16].

However, the implicit function theory only guarantees the existence of the ideal controller $u^*\left(\boldsymbol{x},v\right)$ for system (1), and does not prescribe a technique for constructing it even if the dynamics of the system are well known. In the following, we propose to design an adaptive PID control to construct this unknown ideal implicit controller.

II. Adaptive PID controller design

A. Control law

To develop the control law, we assume that the unknown implicit ideal controller $u^*\left(\boldsymbol{x},v\right)$ can be approximated by a PID controller u_{pid} given as follows:

$$u_{pid} = K_p e(t) + K_I \int_0^t e(\tau)\, d\tau + K_d \frac{de(t)}{dt}, \quad (11)$$

where K_p is the proportional gain, K_I is the integral gain, and K_d is the derivative gain. For convenience, let $\theta = \left[K_p, K_I, K_d\right]^{\mathrm{T}}$ represent the vector of PID controller gains and $\Pi(e) = \left[e(t), \int_0^t e(\tau)\, d\tau, \frac{de(t)}{dt}\right]^{\mathrm{T}}$.

We emphasize again that θ will be adjusted during the control procedure in this study. Hence, we can rewrite (11) as

$$u_{pid}\left(e,\theta\right) = \Pi^{\mathrm{T}}(e)\theta \quad (12)$$

Moreover, we assume that there exists an optimal bounded time varying parameter vector θ_i^* with bounded time derivative such that the ideal control u_i^* fulfil

$$u^* = \Pi^{\mathrm{T}}(e)\theta^* + \varepsilon\left(\boldsymbol{x}\right) \quad (13)$$

where $\varepsilon\left(\boldsymbol{x}\right)$ is the approximation error, θ^* is an unknown ideal parameter vector which minimizes the function $\left|\varepsilon\left(\boldsymbol{x}\right)\right|$.

Before proceeding we need to introduce an assumption about the approximation error $\varepsilon\left(\boldsymbol{x}\right)$.

Assumption 3. The approximation error $\varepsilon\left(\boldsymbol{x}\right)$ in (13) is bounded as $\left|\varepsilon\left(\boldsymbol{x}\right)\right| \le \bar{\varepsilon}$ where $\bar{\varepsilon}$ is a positive constant.

Since the ideal parameter vector θ^* is unknown, so it should be estimated by a suitable adaptation law. Let θ be an estimate of the ideal vector θ^* and define the control law as the adaptive PID approximation of the ideal controller (13), i.e., the control law for system (1) is chosen as

$$u = u_{pid} = \Pi^T(e)\theta \quad (14)$$

After the specification of the controller structure, the next step should be the design of an adaptive law for the free parameters θ such that the control law u approximates, as best as possible, the implicit ideal controller u^* . To this end, a gradient descend adaptation algorithm will be developed in the next subsection.

B. Adaptation law for PID control

Our goal in this subsection is to design an adaptive algorithm for the parameter estimates θ such that the PID controller (14) approximates the unknown ideal controller (13), i.e., the adaptive algorithm should be designed to make the error between u^* and u as small as possible. Furthermore, the adaptive law should guarantee the boundedness of the parameters estimates. To this end, let us define the error between the controllers u^* and u as:

$$e_u = u^* - u \tag{15}$$

The error e_u represents the actual deviation between the unknown function u^* and the control input u_{pid} .

Using (14) and (13), (15) becomes

where $\tilde{\theta} = \theta^* - \theta$ is the parameter estimation error vector.

By invoking the mean value theorem, there exists a constant λ with $0 < \lambda < 1$, such that the nonlinear function $f(\boldsymbol{x},u)$ can be expressed around u^* as

$$f(\boldsymbol{x},u) = f(\boldsymbol{x},u^*) + f_{u_\lambda}(u-u^*) \tag{16}$$

where $f_{u=u_\lambda} = \partial f(\boldsymbol{x},u)/\partial u|_{u=u_\lambda}$ with $u_\lambda = \lambda u + (1-\lambda)u^*$.

By substituting (17) into the error equation (7), we get

$$\begin{aligned}\dot{s} &= -Ks - K_0 \tanh(s/\varepsilon_0) \\ &\quad -\left(f_{u_\lambda}(u-u^*) + f(\boldsymbol{x},u^*) - v\right)\end{aligned} \tag{17}$$

Using (9), (18) becomes

$$\dot{s} = -Ks - K_0 \tanh(s/\varepsilon_0) - f_{u_\lambda}(u-u^*) \tag{18}$$

which can be rewritten in the following form:

$$\dot{s} + Ks + K_0 \tanh(s/\varepsilon_0) = f_{u_\lambda}(u^*-u) = f_{u_\lambda}e_u \tag{19}$$

We notice here that u^* is an unknown quantity, so the signal e_u defined in (15) is not available. Eq. (20) will be used to overcome this difficulty. Indeed, from (20), we see that even if the signal e_u is not available for measurement, the quantity, $f_{u_\lambda}e_u$, is measurable. This fact will be exploited in the design of the parameters adaptive law.

Now, consider a quadratic cost function; that measures the discrepancy between the implicit ideal controller u^* and the actual PID controller u_{pid} , defined as

$$J(\theta) = \frac{1}{2}e_u^2 = \frac{1}{2}(u^*-u)^2 = \frac{1}{2}\left(u^* - \Pi^{\mathrm{T}}(e)\theta\right)^2 \tag{20}$$

The gradient descent method is used here to minimize the cost function (21). Hence, by applying the gradient descent method [16, 19], we obtain as an adaptive law for the parameters θ , the following first order differential equation

$$\dot{\theta} = -\eta(t)\nabla_\theta J(\theta) \tag{21}$$

where $\eta(t)$ is a positive time-varying parameter.

From (21), the gradient of $J(\theta)$ with respect to θ is

$$\frac{\partial J(\theta)}{\partial \theta} = -\Pi(e)e_u \tag{22}$$

Therefore, the gradient descent algorithm becomes

$$\dot{\theta} = \eta(t)\Pi(e)e_u \tag{23}$$

The adaptive law (24) cannot be implemented since the signal e_u is not available. In order to render (24) computable, from Eq. (20), we select the design parameter $\eta(t)$ as $\eta(t) = \eta_0 f_{u\lambda}$, where η_0 is a positive constant. Thus, (24) becomes

$$\dot{\theta} = \eta_0 \Pi(e)\{f_{u\lambda}e_u\} \tag{24}$$

Using (20), we get

$$\dot{\theta} = \eta_0 \Pi(e)\{\dot{s} + Ks + K_0 \tanh(s/\varepsilon_0)\} \tag{25}$$

As shown in [20], the adaptive law (26) cannot guarantee the boundedness of the parameters $\tilde{\theta}$ in the presence of approximation errors that are unavoidable in such adaptive schemes. So, to improve the robustness of the adaptive law (26) in the presence of approximation errors, we modify it by introducing a σ-modification term as follows [20]:

$$\dot{\theta} = \eta_0 \Pi(e)\{\dot{s} + Ks + K_0 \tanh(s/\varepsilon)\} - \eta_0\sigma\theta \tag{26}$$

where σ is a small positive constant. We notice that the adaptive law is modified so that the time derivative of the Lyapunov function used to analyze this adaptive law becomes negative in the space of the parameter estimates when these parameters exceed certain bound [20].

C. Stability of the closed-loop system

In order to analyze the tracking error convergence and the stability of the closed-loop system, let us consider the following Lyapunov-like function

$$V = \frac{1}{2}s^2 + \frac{1}{2\eta_0}\tilde{\theta}^{\mathrm{T}}\tilde{\theta} \tag{27}$$

The time derivative of (28) can be written as

$$\dot{V} = s\dot{s} - \frac{1}{\eta_0}\tilde{\theta}^{\mathrm{T}}\dot{\theta} + \frac{1}{\eta_0}\tilde{\theta}^{\mathrm{T}}\dot{\theta}^* \tag{28}$$

Using (19) and (27), (29) becomes

$$\begin{aligned}\dot{V} &= s\left(-Ks - K_0 \tanh\left(\frac{s}{\varepsilon_0}\right) + f_{u\lambda}e_u\right) - \\ &\quad \tilde{\theta}^{\mathrm{T}}\left(\Pi(e)f_{u\lambda}e_u - \sigma\theta\right) + \frac{1}{\eta_0}\tilde{\theta}^{\mathrm{T}}\dot{\theta}^*\end{aligned} \tag{29}$$

With (16), we can write

$$\begin{aligned}\dot{V} &= -Ks^2 - K_0 s \tanh\left(\frac{s}{\varepsilon_0}\right) + s f_{u\lambda}e_u - \\ &\quad \left(e_u - \varepsilon(\boldsymbol{x})\right)f_{u\lambda}e_u + \sigma\tilde{\theta}^{\mathrm{T}}\theta + \frac{1}{\eta_0}\tilde{\theta}^{\mathrm{T}}\dot{\theta}^*\end{aligned} \tag{30}$$

or

$$\dot{V} = -Ks^2 - K_0 s \tanh\left(\frac{s}{\varepsilon_0}\right) + s f_{u\lambda} e_u - e_u f_{u\lambda} e_u + \varepsilon(\boldsymbol{x}) f_{u\lambda} e_u + \sigma \tilde{\theta}^{\mathrm{T}} \theta + \frac{1}{\eta_0} \tilde{\theta}^{\mathrm{T}} \dot{\theta}^* \quad (31)$$

Using inequalities

$$\sigma \tilde{\theta}^{\mathrm{T}} \theta \le -\frac{\sigma}{2} \left\|\tilde{\theta}\right\|^2 + \frac{\sigma}{2} \left\|\theta^*\right\|^2 \quad (32)$$

$$\varepsilon(\boldsymbol{x}) f_{u_\lambda} e_u \le \frac{1}{4} f_{u_\lambda} e_u^2 + f_{u_\lambda} \varepsilon^2(\boldsymbol{x}) \quad (33)$$

$$s f_{u\lambda} e_u \le \frac{1}{4} f_{u\lambda} e_u^2 + f_{u\lambda} s^2 \quad (34)$$

equation (32) can be bounded as

$$\dot{V} \le -\frac{1}{2} f_{u_\lambda} e_u^2 - K_0 s \tanh\left(\frac{s}{\varepsilon_0}\right) - \left(K - f_{u\lambda}\right) s^2 - \frac{\sigma}{4} \left\|\tilde{\theta}\right\|^2 + \frac{\sigma}{2} \left\|\theta^*\right\|^2 + \frac{1}{\sigma {\eta_0}^2} \left\|\dot{\theta}^*\right\|^2 + f_{u\lambda} \varepsilon^2(\boldsymbol{x}) \quad (35)$$

Since the parameters $\theta^*(t)$ and $\dot{\theta}^*(t)$, the functions $\varepsilon(\boldsymbol{x})$ and f_{u_λ} are assumed bounded in this paper, so we can define a positive constant bound ψ as

$$\psi = \sup_t \left(\frac{\sigma}{2} \left\|\theta^*\right\|^2 + \frac{1}{\sigma {\eta_0}^2} \left\|\dot{\theta}^*\right\|^2 + f_{u_\lambda} \varepsilon^2(\boldsymbol{x}) \right) \quad (36)$$

Then, (36) can be simplified to

$$\dot{V} \le -\frac{1}{2} f_{u_\lambda} e_u^2 - K_0 s \tanh\left(\frac{s}{\varepsilon_0}\right) - \left(K - f_{u\lambda}\right) s^2 - \frac{\sigma}{4} \left\|\tilde{\theta}\right\|^2 + \psi \quad (37)$$

Assuming that the design parameter K is chosen such that $K > \delta_1$, and $\gamma = \min\left(2 \times \left(K - \delta_1\right), 0.5\, \sigma \eta_0\right)$, the inequality (36) can be written as follows

$$\dot{V} \le -\frac{1}{2} f_{u_\lambda} e_u^2 - K_0 s \tanh\left(\frac{s}{\varepsilon_0}\right) - \frac{\gamma}{2} s^2 - \frac{\gamma}{2\eta_0} \left\|\tilde{\theta}\right\|^2 + \psi \quad (38)$$

or,

$$\dot{V} \le -\gamma V + \psi \quad (39)$$

Now we can prove the following theorem that shows the boundedness of all variables in the closed-loop system.

Theorem 2: *Consider the system (1). Suppose that Assumptions 1-3 are satisfied. Then the control law defined by (14) with the adaptation law given by (27) guarantees that the closed-loop system is UUB stable and the output tracking error converges to a small neighborhood of the origin.*

Proof : From (40), we can have

$$V(t) \le V(0) e^{-\gamma t} + \frac{\psi}{\gamma} \quad (40)$$

Then from (41), it can be shown that for $V \ge \psi/\gamma$ we have $\dot{V} < 0$. According to a standard Lyapunov theorem, the signals $s(t)$, $\tilde{\theta}(t)$ and $u(t)$ in the closed-loop system are bounded. Moreover, from (28) and (41) we can write $\left|s(t)\right| \le \sqrt{\left|s(0)\right|^2 + \frac{1}{\eta_0}\left|\tilde{\theta}(0)\right|^2} e^{-0.5\gamma t} + \sqrt{\frac{2\psi}{\gamma}}$, and in order to achieve the tracking error convergence to a small neighborhood around zero, the parameters K , σ and η_0 should be chosen appropriately. Then, it is possible to make $\sqrt{\frac{2\psi}{\gamma}}$ as small as desired. Denotes $\Phi = \sqrt{\frac{2\psi}{\gamma}}$, since $e^{-0.5\gamma t} \to 0$ as $t \to \infty$, it exists T such that $\left|s(t)\right| \le \Phi$ for $t > T$. This implies that the tracking errors converge to residual sets as: $\left\|e^{(i)}(t)\right\| \le 2^i \lambda^{i-n+1} \Phi$, $i = 0, \ldots, n-1$. This completes the proof.

Remark 2: It is worth to notice that in the PID controller (14) there is no robustifying control term. In this paper, the term $K_0 \tanh\left(s/\varepsilon_0\right)$ in the parameter adaptive law plays, in some way, the role of a robustifying term. Hence the robustness of the controller can be improved by selecting large positive values for the design parameters K_0.

Remark 3: Because the aim of the σ-modification adaptive law (7) is to avoid parameter drift, it does not need to be active when the estimated parameters are within some acceptable bound. Therefore, a more reasonable modification would be to select σ as [20]: $\sigma = 0$, if $\left\|\theta\right\| \le M_\theta$, $\sigma = \sigma_0$, otherwise; where M_θ and σ_0 are design positive constants, and $M_\theta \ge \sup_{t \ge 0}\left(\left\|\theta^*\right\|\right)$.

III. Simulation results

In order to illustrate previous results, an application of the proposed control scheme to a SISO nonaffine system is displayer in the sequel. The nonaffine system is described by the following differential equation [18]

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_1^2 + 0.15u^3 + 0.1\left(1 + x_2^2\right)u + \sin\left(0.1u\right) \\ \quad + d(t) \end{cases} \quad (41)$$

where $d(t)$ is an external disturbance included in order to test the robustness of the adaptive PID controller against external disturbances.

Let us define $y(t)=x_1(t)$, $\boldsymbol{x}=[x_1,x_2]^{\mathrm{T}}$, $f(\boldsymbol{x},u)=x_1^2+0.15u^3+0.1(1+x_2^2)u+\sin(0.1u)+d(t)$. Then, the system given by (42) can be expressed as

$$\ddot{y}=f(\boldsymbol{x},u) \qquad (42)$$

which is in the input-output form given by **Erreur ! Source du renvoi introuvable.**. Such that, the nonlinear function $f(\boldsymbol{x},u)$ is assumed completely unknown, i.e., the control system does not require the knowledge of the system's model as in conventional model-based adaptive controllers. In fact, the dynamic model of the system is only required for simulation purposes.

The control objective is to force the system output $y(t)=x_1(t)$ to track the desired trajectory $y_d(t)=\sin(t)+\cos(0.5t)$. The system initial conditions are $\boldsymbol{x}(0)=[0.6,0.5]^{\mathrm{T}}$, and the initial values of the parameter estimates $\theta(0)$ are set equal to zero. The design parameters used in this simulation are chosen as follows: $\lambda=15$, $K=5$, $K_0=40$, $\varepsilon_0=0.01$, $\eta_0=5$, and $\sigma=0.02$. A Gaussian white noise with mean zero and variance 0.02 is added to the measurement $y=x_1$.

Within this simulation, the unknown ideal implicit controller is approximated by a PID controller in the form of **Erreur ! Source du renvoi introuvable.** with an adaptive law in the form of **Erreur ! Source du renvoi introuvable.**. The desired and actual outputs are illustrated in figure 1. The control input signal is shown in figure 2, and the evolution of the gains K_p, K_I and K_d are given in figures 3, 4 and 5, respectively. We can note that the actual trajectory converges to the desired trajectory, the control signal is almost smooth and the parameter estimates are bounded. These simulation results demonstrate the tracking capability of the proposed adaptive PID controller and its effectiveness for control tracking of uncertain nonaffine nonlinear systems.

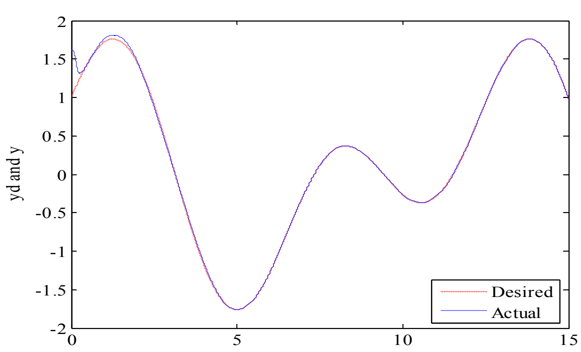

Figure 1. Desired and actual outputs

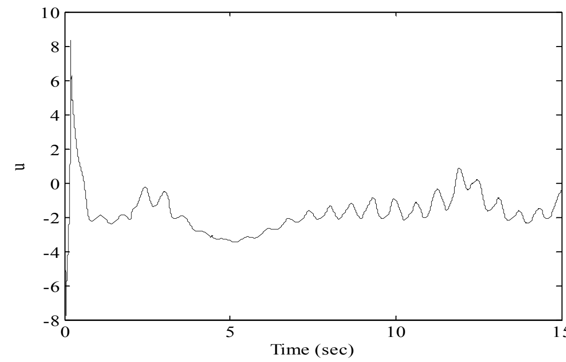

Figure 2. Control input signal

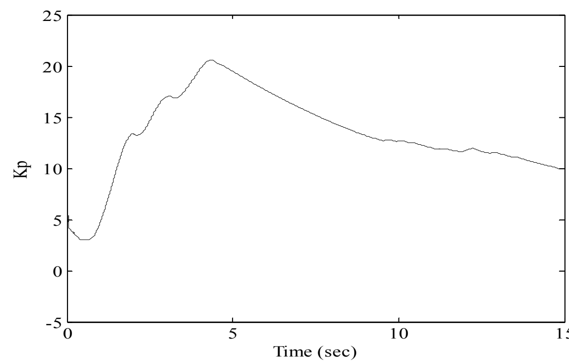

Figure 3. Evolution of the gains K_p

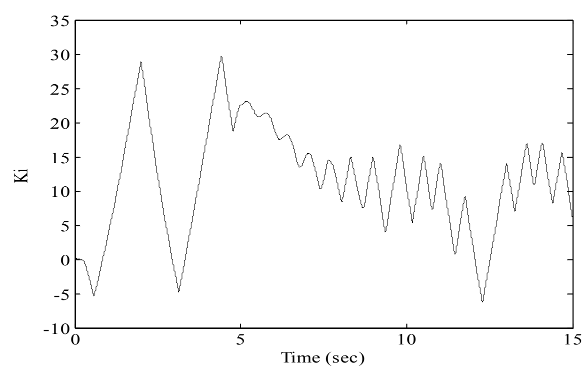

Figure 4. Evolution of the gains K_I

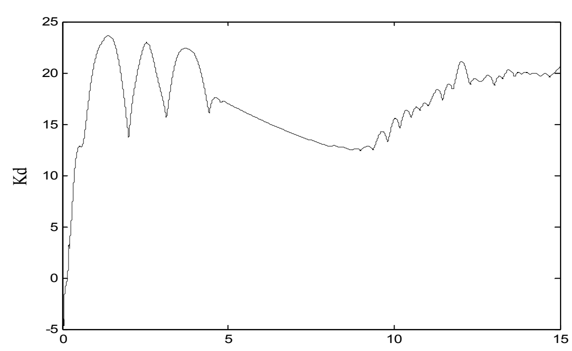

Figure 5. Evolution of the gains K_d

IV. CONCLUSION

In this paper, we proposed a stable self-tuning PID control scheme for a class of SISO nonaffine nonlinear systems. The scheme consists of an adaptive PID controller with its adaptive law. The PID algorithm is used to construct adaptively an unknown ideal implicit controller, and its adjustable parameters are updated, by using the gradient descent method, in order to minimize

the error between the unknown controller and the used PID controller. The proposed control scheme does not require the knowledge of the mathematical model of the plant, guarantees the boundedness of all the signals in the closed-loop system, and ensures the convergence of the tracking errors to a neighborhood of the origin. Simulation results show the good performances of the proposed controller. Future work will focus on improvement of the proposed controller by introducing a state observer to provide an estimate of the state vector.

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