

# Observer-based parametric fault estimation for a class of Lipschitz nonlinear systems

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**Abstract** - In this paper, an adaptive observer-based approach for joint estimation of states and parametric faults is introduced for a class of Lipschitz nonlinear systems. The novelty of the proposed approach is that instead of the observer matching condition, it relies on a new necessary condition for the existence of the adaptive observer. Under a relaxed Lipschitz condition the problem of designing the adaptive observer is formulated into the basic feasibility problem of linear matrix inequalities. Finally, an illustrative example with simulation results is provided to verify the efficiency of the proposed approach.

**Index Terms** - Nonlinear observer, adaptive observer, parametric faults, fault estimation, observer matching condition, LMI.

## I. INTRODUCTION

The growing complexity and automation degree of modern technical processes increase the possibility of system failures. Faults in sensors, actuators or process components may lead to the degradation of the overall system performance and could cause disastrous results as the system may go unstable. Therefore, to improve system's reliability and avoid performance deterioration or system shutdown, faults have to be detected and localised timely while the system is still operating in a controllable region. This has stimulated over the last few decades an intense interest in the development of fault detection and identification (FDI) methods.

One of the most used schemes in this area is the so-called observer-based FDI technique. The basic idea behind the use of observers for FDI is to generate fault indicator signals, called the residuals, as a weighted difference between the estimated outputs and the measured outputs [1][2]. One can determine whether the system is suffering from some faults or not by comparing the residual with a fixed or adaptive threshold. So far, various observer-based FDI design approaches have been reported in the literature (see survey papers and references therein).

During the last decade, motivated by the problem of active fault tolerant control, there has been a great interest on fault reconstruction and estimation (FRE) [3].

FRE is different from the majority of FDI methods in the sense that it provides estimates of faults. Various FRE design approaches have been reported in the literature mainly based on sliding mode observers and adaptive observers. Edwards and al. proposed an approach based on a sliding mode observer (SMO) where the sliding motion was maintained even in the presence of faults which can be reconstructed under certain conditions [4]. Later it was extended in [5] where sensor faults were considered. For nonlinear Lipschitz systems, in [6]-[8] sliding mode observer-based robust fault reconstruction were addressed by assuming that disturbances are matched. More recently, a sliding mode observer-based fault reconstruction approach with  $H_\infty$  performance was proposed in [9].

Adaptive observers have been used for fault estimation by different authors [10]-[14]. However they assume that faults can be lumped into the so-called observer matching condition which is not satisfied for some practical systems. In order to remove the observer matching condition, two adaptive observer-based fault estimation methods were developed in [11]. But the gain of the designed observer is dependent on the fault distribution matrix. A different technique was provided in [15] where a high gain adaptive observer for joint estimation of the state vector and the parameter vector related to faults was developed.

Motivated by the work in [16] and [17], in this paper, an approach for parametric fault estimation is proposed using an adaptive observer technique for a class of nonlinear Lipschitz systems. The main contributions of this work are twofold. First, unlike previous works, relative degrees between faults and outputs considered here are two. This means that the observer matching condition is not verified. And we give the necessary condition for the existence of such adaptive observer. Second, the adaptive law is much simpler than in [16] and [17] where the proposed adaptation laws rely on partial differential equation in the first and a set of matrix equalities in the second.

## II. PROBLEM STATEMENT AND STRUCTURAL ASSUMPTIONS

Consider the uncertain nonlinear system governed by the following set of differential equations

$$\begin{cases} \dot{x} = Ax + E(\Phi(x, u) + \Psi(x, u))\theta \\ y = Cx \end{cases} \quad (1)$$

where  $x \in R^n$  is the state vector,  $u \in R^m$  is the control input vector,  $y \in R^p$  is the output vector and  $\theta \in R^q$  is an unknown vector that encapsulates parameters used to model the effect of possible multiplicative faults and parametric uncertainties.  $A \in R^{n \times n}$ ,  $C \in R^{p \times n}$  and  $E \in R^{n \times r}$  are known constant matrices.  $\Phi: R^n \times R^m \rightarrow R^r$  and  $\Psi: R^n \times R^m \rightarrow R^{r \times q}$  are known smooth nonlinear functions.

It is assumed that  $(A, C)$  is an observable pair and the control signals are bounded smooth known functions and ensure that the state  $x$  remains bounded even when a fault occurs. Furthermore, without loss of generality,  $C$  and  $E$  are assumed to be full rank.

The adaptive observer-based fault estimation problem for system (1) is to construct an observer that can estimate jointly  $x$  and  $\theta$ . Most available methods for adaptive observer design assume that the so-called observer matching condition holds, i.e. there exist a matrix  $F \in R^{r \times p}$  and a symmetric positive definite matrix  $P \in R^{n \times n}$  such that

$$E^T P = FC \quad (2)$$

It is convenient to verify the feasibility of (2) using the following lemma.

**Lemma 1** [18] *There exist matrices  $P = P^T$  and  $F$  verifying  $E^T P = FC$ , if and only if the condition  $\text{rank}(CE) = \text{rank}(E)$  is satisfied.*

One can easily see that the observer matching condition means that  $r \leq p$  and the relative degrees from the unknown parameters to at least  $q$  measured outputs are all one. For many physical systems modelled by (1), the observer matching condition is not satisfied. For instance, for Lagrangian systems in many cases only the positions are measurable, and thus, none

of possible parametric faults or parametric uncertainties can be found in the measured dynamics.

The main purpose of this paper is to design an adaptive observer for system (1) when the observer matching condition is not met. Here the structure of the system is defined by the following assumption.

**Assumption 1** *Matrices  $A, C$  and  $E$  satisfy*

$$CE = 0 \quad (3)$$

$$\text{rank}(CAE) = \text{rank}(E) \quad (4)$$

Assumption 1 implies that the relative degrees from the unknown parameters to at least  $r$  measured outputs are two. We need this structural condition to ensure the realizability of the proposed adaptation law for the parameters estimate. Although this assumption looks to be too restrictive, it can be satisfied by many physical systems like mechanical systems, quadrotors and the drilling system [16].

In the subsequent analysis, the following assumptions will be imposed on system (1).

**Assumption 2** *The vector  $\theta$  is piecewise constant and bounded in the following sense*

$$\|\theta\| \leq \rho \quad (5)$$

where  $\rho$  is a known positive constant.

**Assumption 3** *The nonlinear function vector  $\Phi(x, u)$  and the matrix  $\Psi(x, u)$  satisfy the conditions*

$$\|\Phi(x, u) - \Phi(\hat{x}, u)\| \leq \|H_1(x - \hat{x})\| \quad (6)$$

$$\|\Psi(x, u) - \Psi(\hat{x}, u)\| \leq \|H_2(x - \hat{x})\| \quad (7)$$

$\forall x, \hat{x} \in R^n, \forall u \in R^m$ . Furthermore  $\Psi(x, u)$  and its time derivative are globally bounded.

**Assumption 4** *The matrix  $E\Psi(x, u)$  is persistently exciting i.e. there exist  $T, k_1, k_2 > 0$  such that for all  $t \geq 0$*

$$I_n k_1 \geq \int_t^{t+T} E\Psi(x, u)\Psi(x, u)^T E^T \geq k_2 I_n \quad (8)$$

Assumption 2 shows that there is an a priori knowledge on the fault parameter vector  $\theta$ . In practice the bound  $\rho$  can be set by exploiting some knowledge on the physical nature of the faults. Assumption 3 is a less conservative Lipschitz condition. It was reported in [19] **Erreur! Source du renvoi introuvable.** that  $\|H_1(x - \hat{x})\|$  can be much smaller than its counterpart with a Lipschitz constant for the same nonlinear function. This can be significantly helpful in finding a solution to the problem of observer design when the Lipschitz constant is large. The persistency of excitation condition in assumption 4 is crucial for ensuring parameter identification. In fact, it guarantees the observability of the unknown parameter vector. This fact can be understood through the Gramian observability matrix of the augmented system obtained

by combining the unknown parameter vector with the state vector.

### III. ADAPTIVE OBSERVER DESIGN

#### A. Canonical form for adaptive observer design

Decomposing  $C$  and  $E$  into bloc matrices as follows

$$[C_1 \quad C_2], \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \quad (9)$$

where  $C_1 \in R^{p \times p}$  and  $E_1 \in R^{p \times r}$ . Without loss of generality, it can be assumed that  $C_1$  is full rank. Define new coordinates as  $z = Tx$  where

$$T = \begin{bmatrix} C_1 & C_2 \\ 0_{(n-p) \times p} & I_{(n-p)} \end{bmatrix} \quad (10)$$

Then, in the new coordinate system, the original system (1) has the following form

$$\begin{cases} \dot{z} = \bar{A}z + \bar{E}(\Phi(T^{-1}z, u) + \Psi(T^{-1}z, u)\theta) \\ y = \bar{C}z \end{cases} \quad (11)$$

where

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, y = z_1 T^{-1} = \begin{bmatrix} C_1^{-1} & -C_1^{-1}C_2 \\ 0_{(n-p) \times p} & I_{(n-p)} \end{bmatrix},$$

$$\bar{A} = TAT^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \bar{E} = TE = \begin{bmatrix} 0_{p \times r} \\ E_2 \end{bmatrix},$$

$$\bar{C} = [I_{p \times p} \quad 0_{p \times (n-p)}].$$

Considering the structure of  $\bar{E}$ , we conclude that  $\text{rank}(E_2) = r$ .

Applying equality (4) to the system (11) yields

$$\text{rank}(A_{12}E_2) = \text{rank}(E_2) \quad (12)$$

This equality has the form of the well-known observer matching condition. It implies that  $n - p \geq r$ . The following lemma is a straightforward result of Lemma 1.

**Lemma 2** *The equality  $\text{rank}(A_{12}E_2) = \text{rank}(E_2)$  holds if and only if there exist matrices  $P_3 = P^T > 0$  and  $F$  such that*

$$E_2^T P_3 = FA_{12} \quad (13)$$

#### B. Main result

Based on the transformed system (11), we propose the following adaptive observer

$$\dot{\hat{z}} = \bar{A}\hat{z} + \bar{E}(\hat{\Phi} + \hat{\Psi}\hat{\theta}) + \bar{L}(y - \bar{C}\hat{z}) \quad (14)$$

$$\hat{\theta} = W(\hat{z}, u) + \Gamma\hat{\Psi}^T Fy \quad (15)$$

$$\dot{\hat{\theta}} = -\Gamma \frac{d\hat{\Psi}^T}{dt} Fy - \Gamma\hat{\Psi}^T (FA_{11}y + FA_{12}\hat{z}_2 - E_2^T P_2^T \hat{z}_1) \quad (16)$$

where  $\hat{\Phi} = \Phi(T^{-1}\hat{z}, u)$ ,  $\hat{\Psi} = \Psi(T^{-1}\hat{z}, u)$ ,  $\Gamma = \Gamma^T > 0$  is the learning rate matrix and  $P_2$  is a matrix to be designed later. Letting  $\tilde{z} = z - \hat{z}$ ,  $\tilde{\Phi} = \Phi(T^{-1}z, u) - \hat{\Phi}$

and  $\tilde{\Psi} = \Psi(T^{-1}z, u) - \hat{\Psi}$ , then the error dynamical equations can be characterized as

$$\dot{\tilde{z}} = (\bar{A} - \bar{L}\bar{C})\tilde{z} + \bar{E}(\tilde{\Phi} + \tilde{\Psi}\theta + \hat{\Psi}\tilde{\theta}) \quad (17)$$

$$\dot{\tilde{\theta}} = -\Gamma\hat{\Psi}^T [E_2^T P_2^T \quad FA_{12}] \tilde{z} \quad (18)$$

**Theorem 1** *Under Assumptions 1-4 the error dynamical system (17),(18) is asymptotically stable if there exist positive real numbers  $\varepsilon_1, \varepsilon_2$  and matrices  $P = P^T > 0$  and  $F$ , such that*

$$P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix} > 0 \quad (19)$$

$$(\bar{A} - \bar{L}\bar{C})P + P(\bar{A} - \bar{L}\bar{C})^T + (\varepsilon_1 + \varepsilon_2)P\bar{E}\bar{E}^T P + \varepsilon_1^{-1}\bar{H}_1^T \bar{H}_1 + \varepsilon_2^{-1}\bar{H}_2^T \bar{H}_2 < 0 \quad (20)$$

$$E_2^T P_3 = FA_{12} \quad (21)$$

where  $\bar{H}_1 = H_1 T^{-1}$  and  $\bar{H}_2 = H_2 T^{-1}$ .

*Proof.* To prove the result in Theorem 1, consider the following Lyapunov function

$$V = \tilde{z}^T P \tilde{z} + \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \quad (22)$$

The derivative of  $V$  along with the trajectories of error dynamic systems (17),(18) is

$$\dot{V} = \tilde{z}^T [(\bar{A} - \bar{L}\bar{C})P + P(\bar{A} - \bar{L}\bar{C})^T] \tilde{z} + 2\tilde{z}^T P\bar{E}[\tilde{\Phi} + \tilde{\Psi}\theta] + 2\tilde{z}^T P\bar{E}\tilde{\theta} - 2\tilde{\theta}^T \hat{\Psi}^T [E_2^T P_2^T \quad FA_{12}] \tilde{z} \quad (23)$$

By using the Young's inequality  $v^T w \leq \varepsilon v^T v + w^T w$  ( $\varepsilon > 0$ ) and the Lipschitz conditions (6),(7), we obtain the following inequalities:

$$2\tilde{z}P\bar{E}\tilde{\Phi} \leq \varepsilon_1 \tilde{z}^T P\bar{E}\bar{E}^T P \tilde{z} + \tilde{z}^T \varepsilon_1^{-1} \bar{H}_1^T \bar{H}_1 \tilde{z}$$

$$2\tilde{z}P\bar{E}\tilde{\Psi}\theta \leq \varepsilon_2 \tilde{z}^T P\bar{E}\bar{E}^T P \tilde{z} + \tilde{z}^T \rho^2 \varepsilon_2^{-1} \bar{H}_2^T \bar{H}_2 \tilde{z}$$

Where  $\bar{H}_1 = H_1 T^{-1}$  and  $\bar{H}_2 = H_2 T^{-1}$ . Then the derivative of the Lyapunov function satisfies the following inequality

$$\dot{V} \leq \tilde{z}^T [(\bar{A} - \bar{L}\bar{C})P + P(\bar{A} - \bar{L}\bar{C})^T + (\varepsilon_1 + \varepsilon_2)P\bar{E}\bar{E}^T P + \varepsilon_1^{-1}\bar{H}_1^T \bar{H}_1 + \rho^2 \varepsilon_2^{-1}\bar{H}_2^T \bar{H}_2] \tilde{z} + 2\tilde{z}^T P\bar{E}\tilde{\theta} - 2\tilde{\theta}^T \hat{\Psi}^T [E_2^T P_2^T \quad FA_{12}] \tilde{z} \quad (24)$$

Using the decomposed structures of  $P$  and  $\bar{E}$  and equality (13), we get

$$\dot{V} \leq \tilde{z}^T [(\bar{A} - \bar{L}\bar{C})P + P(\bar{A} - \bar{L}\bar{C})^T + (\varepsilon_1 + \varepsilon_2)P\bar{E}\bar{E}^T P + \varepsilon_1^{-1}\bar{H}_1^T \bar{H}_1 + \rho^2 \varepsilon_2^{-1}\bar{H}_2^T \bar{H}_2] \tilde{z} + 2\tilde{z}^T \begin{bmatrix} P_2 E_2 \\ P_3 E_2 \end{bmatrix} P\bar{E}\tilde{\theta} - 2\tilde{\theta}^T \hat{\Psi}^T [E_2^T P_2^T \quad FA_{12}] \tilde{z} = \tilde{z}^T (\bar{A} - \bar{L}\bar{C})P + P(\bar{A} - \bar{L}\bar{C})^T + (\varepsilon_1 + \varepsilon_2)P\bar{E}\bar{E}^T P + \varepsilon_1^{-1}\bar{H}_1^T \bar{H}_1 + \rho^2 \varepsilon_2^{-1}\bar{H}_2^T \bar{H}_2] \tilde{z} \quad (25)$$

Therefore,  $\dot{V} < 0$  if (20) is satisfied. So based on Lyapunov stability theory, we conclude that the equilibrium  $\tilde{z} = 0, \tilde{\theta} = 0$  is stable. Now to prove the asymptotic stability of the observer, we write

$$Q = -[(\bar{A} - \bar{L}\bar{C})P + P(\bar{A} - \bar{L}\bar{C})^T + (\varepsilon_1 + \varepsilon_2)P\bar{E}\bar{E}^T P + \varepsilon_1^{-1}\bar{H}_1^T\bar{H}_1 + \rho^2\varepsilon_2^{-1}\bar{H}_2^T\bar{H}_2] \quad (26)$$

Substituting (26) into (25), yields

$$\dot{V} \leq \tilde{z}^T Q \tilde{z} \quad (27)$$

Integrating both sides of inequality (27) from  $t = 0$  to  $t = t_f$ , it follows that

$$V(t_f) \leq V(0) - \int_0^{t_f} \tilde{z}^T Q \tilde{z} dt \quad (28)$$

Since  $V > 0$ , the above inequality implies that

$$\int_0^{t_f} \tilde{z}^T Q \tilde{z} dt \leq V(0) \quad (29)$$

So for  $t_f \rightarrow +\infty$  the above integral is finite and always less than or equal  $V(0)$ . By using the Barbalat's lemma [20], it follows that  $\lim_{t \rightarrow +\infty} \tilde{z}^T Q \tilde{z} = 0$  and thus  $\lim_{t \rightarrow +\infty} \tilde{z} = 0$ . Consequently, it can also be concluded that  $\lim_{t \rightarrow +\infty} \dot{\tilde{z}} = 0$ . Also, since both  $\Phi(T^{-1}z, u)$  and  $\Psi(T^{-1}z, u)$  are Lipschitz, considering (17), we have

$$\lim_{t \rightarrow +\infty} \bar{E}\hat{\Psi}\tilde{\theta} = 0 \quad (30)$$

Thus if the persistency excitation condition (6) holds, the fault estimates converge to their true values.

### C. An LMI approach to compute $P, L$ and $F$

Adaptive observer design by using Theorem 1 involves solving inequalities (19),(20) under the constraint (21) for  $P, L$  and  $F$ . If we put  $L = P^{-1}M\bar{C}^T$  where  $M = M^T$  and using the Schur complement [21], inequality (20) can be modified into the following LMI problem

$$\begin{bmatrix} \Lambda & P\bar{E} & P\bar{E} \\ \bar{E}^T P & \varepsilon_1^{-1}I_r & 0_{r \times r} \\ \bar{E}^T P & 0_{r \times r} & \varepsilon_2^{-1}I_r \end{bmatrix} < 0 \quad (31)$$

where  $\Lambda = A^T P + PA - C^T C M - M C^T C + \varepsilon_1^{-1}\bar{H}_1^T\bar{H}_1 + \rho^2\varepsilon_2^{-1}\bar{H}_2^T\bar{H}_2$ .

Inequalities (19) and (31) can be solved using LMI solvers, but solving difficulty is added because of the constraint (21). We can show that equality (21) holds if and only if the following optimization problem has a minimum of  $\eta = 0$  [18].

Minimise  $\eta$  subject to

$$\begin{bmatrix} \eta I_r & E_2^T P_3 - FA_{12} \\ (E_2^T P_3 - FA_{12})^T & \eta I_r \end{bmatrix} \quad (32)$$

where  $\eta$  is a positive scalar. Therefore, computing  $P, L$  and  $F$  involves solving (19),(31) and (32), for  $P, M, F$  and  $\eta$ , simultaneously.

## IV. NUMERICAL EXAMPLE

Consider the model of a single-link flexible-joint robotic manipulator defined by the following nonlinear system equations [14], [19].

$$\begin{aligned} \dot{\theta}_m &= \omega_m \\ \dot{\omega}_m &= \frac{k}{J_m}(\theta_l - \theta_m) - \frac{B}{J_m}\omega_m + \frac{k_\tau}{J_m}u \\ \dot{\theta}_l &= \omega_l \\ \dot{\omega}_l &= -\frac{k}{J_l}(\theta_l - \theta_m) - \frac{mgh}{J_l}\sin(\theta_l) \end{aligned} \quad (33)$$

where  $\theta_m$  and  $\theta_l$  are the angular positions of the motor shaft and the link, respectively, with  $\omega_m$  and  $\omega_l$  being their angular velocities,  $u$  represents the control torque of the motor,  $J_m$  the inertia of the motor,  $J_l$  the inertia of the link,  $k$  the elastic constant,  $m$  the link mass,  $g$  the gravity,  $h$  the centre of mass,  $k$  the amplifier gain and  $B$  the viscous friction coefficient. We assume  $\theta_m$  and  $\theta_l$  to be measurable as outputs. The parameters of the system are typical and are taken from [22].

In order to evaluate the effectiveness of the proposed method, we introduce, at  $t = 5$  seconds, an abnormal decrease of 20% of the elastic constant and an abnormal increase of 30% of viscous friction. This leads to the fault vector  $= [-0.036 \ 0.0017]^T$  at  $t = 5$  seconds. Thus, considering the state  $z = [z_1 \ z_2 \ z_3 \ z_4]^T = [\theta_m \ \theta_l \ \omega_m \ \omega_l]^T$ , we can write (33) in the form of (11) with

$$\begin{aligned} \bar{A} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -48.67 & 48.67 & -2.24 & 0 \\ 19.35 & -19.35 & 0 & 0 \end{bmatrix}, \bar{E} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ \bar{C} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \Phi = \begin{bmatrix} 21.62u \\ -33.32 \sin(z_3) \end{bmatrix}, \\ \Psi &= \begin{bmatrix} 270.27(z_2 - z_1) & -270.27z_3 \\ -107.53(z_2 - z_1) & 0 \end{bmatrix}. \end{aligned}$$

By solving the LMI (19),(31) and (32) simultaneously for

$$\begin{aligned} \bar{H}_1 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -33.32 & 0 \end{bmatrix}, \\ \bar{H}_2 &= \begin{bmatrix} -270.27 & 270.27 & -270.27 & 0 \\ 107.53 & -107.23 & 0 & 0 \end{bmatrix}, \varepsilon_1 = 0.5 \\ , \varepsilon_2 &= 150, \rho = 0.091, \text{ we obtain} \end{aligned}$$

$$P = \begin{bmatrix} 399.1384 & 6.4993 & -2.5699 & -0.0863 \\ 6.4993 & 332.6661 & 0.0166 & -1.3565 \\ -2.5699 & 0.0166 & 0.0651 & -0.0029 \\ -0.0863 & -1.3565 & -0.0029 & 0.0866 \end{bmatrix},$$

$$F = \begin{bmatrix} 0.0651 & -0.0029 \\ -0.0029 & 0.0866 \end{bmatrix},$$

$$M = \begin{bmatrix} 657.5 & 375.1 & -61.2 & -10.2 \\ 375.1 & 0 & 10.4 & 0 \\ -61.2 & 10.4 & 1403.7 & 321.3 \\ 10.2 & 0 & 321.3 & 0 \end{bmatrix}.$$

Figures 1-2 exhibit the parametric faults (the dashed lines) and their estimated trajectories (the solid lines) for zero initial conditions and  $\Gamma = 30I_2$ . One can see that faults are accurately estimated.

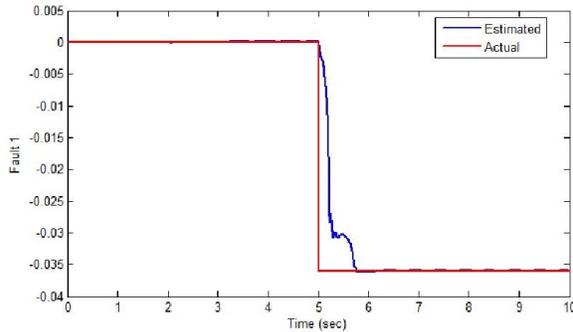


Fig. 1: Actual and observer estimated fault 1

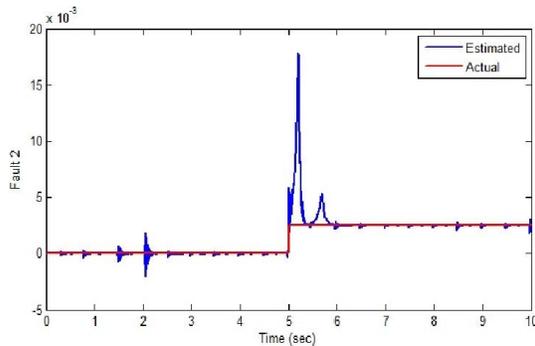


Fig. 2: Actual and observer estimated fault 2

## V. CONCLUSION

In this paper, an adaptive observer for unmatched parametric faults estimation has been proposed for a class of Lipschitz nonlinear systems. The design procedure has been formulated into the basic feasibility problem of strict linear matrix inequalities and has been illustrated by a numerical example, and efficiency has also been demonstrated by the simulations. The proposed method is only applicable to systems with relative degrees being two, between faults and outputs.

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