Direct Adaptive Backstepping Control with Tuning Functions for a Single-Link Flexible-Joint Robot

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Abstract **– In this paper, direct adaptive backstepping control with tuning functions approach for a single-link flexible-joint robot model is proposed. The proposed approach of adaptation is based on the tracking error based parameter adaptation law. First, the direct adaptive backstepping control with tuning functions is applied for a class of nonlinear systems in parametric strict-feedback form to avoid overparametrization. Next, the main steps of the controller design for a single-link flexible-joint robot manipulator model are described. The stability of the proposed controller is studied by using the Lyapunov functions. Finally, the simulation results are given to demonstrate the performance of the proposed approach.**

Index Terms – Single-link flexible-joint robot, backstepping control, direct adaptive control, tuning functions, direct adaptation.

I. Introduction

Backstepping [1-2] is a recursive design procedure for systematically selecting the control Lyapunov function that allows the design of nonlinear controllers for nonlinear systems in strict-feedback form. The idea of backstepping is to design a controller recursively by considering some of the state variables as virtual controls and designing for them intermediate control laws.

Adaptive backstepping is a nonlinear control design technique that has been developed in [3] as an alternative method for the adaptive control of the nonlinear systems, which achieves boundedness of the closed-loop states and convergence of the tracking error to zero. This technique achieves the control of nonlinear systems with parametric uncertainties, these uncertainties consist of unknown constant parameters which appear linearly in the system equations.

The adaptive backstepping design employs more than one estimate per unknown parameter. This overparametrization makes the control law complicated and difficult to implement. The tuning functions are introduced to reduce the dynamic order of the adaptive controller to its minimum. The number of parameter estimates is equal to the number of unknown parameters [1-2].

The adaptive backstepping control method proposed in this paper is a direct adaptive backstepping control with tuning functions method where controller parameters are updated by the tracking error where the parameter adaptation law is driven by tracking error to achieve better parameter estimation and hence better tracking performance.

During the last years, the study of the control of robots manipulators with flexible joints drew a considerable attention [4-5]. The backstepping control design procedure has been used for synthesizing adaptive controllers for a class of flexible-joint robotic manipulators.

In this paper, we propose a parameter adaptation law that is based on the tracking error based adaptation law. The stability of the proposed controller is studied by using the Lyapunov stability theorem and the simulation results are given to demonstrate the performance of the proposed controller.

The paper is organized as follows. In section II, a dynamic nonlinear model for a single-link flexible-joint robot is described. Based on this nonlinear model, direct adaptive backstepping control with tuning functions is designed, and the closed-loop stability analysis is carried out in section III. Simulations results showing the good performance of the proposed controller are presented in section IV. In section V, some conclusions close the paper.

II. Dynamic Modeling of a Single-Link Flexible-Joint Robot

The single-link flexible-joint robot dynamic model is given as follow [4, 6-7]

$$\begin{aligned} J_1\ddot{q}_1 + MgL\sin\left(q_1\right) + K\left(q_1 - q_2\right) &= 0 \\ J_2\ddot{q}_2 - K\left(q_1 - q_2\right) &= u \end{aligned} \tag{1}$$

where u is the input torque. J_1 and J_2 are the inertias of the link and the motor, respectively. M is the link mass and g is the gravity. L is the link length and K is the stiffness. q_1 and q_2 are the angular position of the link and the motor shaft, respectively. The single-link flexible-joint robot is presented schematically in Fig. 1.

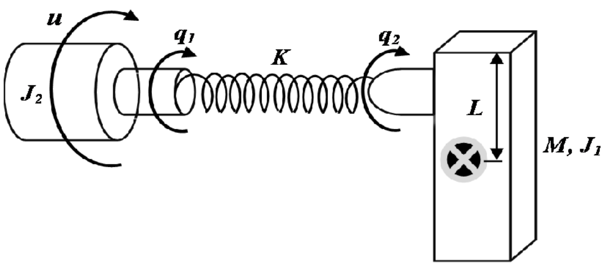

Fig. 1. Single link flexible joint robot.

Let the state variables defined as follows: $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$ and $x_4 = \dot{q}_2$, and its dynamic model becomes

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1(x_1, x_3) + g_1 x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2(x_1, x_3) + g_2 u \end{aligned} \tag{2}$$

with

$$\begin{aligned} f_1(x_1, x_3) &= -\frac{MgL}{J_1}\sin(x_1) - \frac{K}{J_1}x_1, g_1 = \frac{K}{J_1} \\ f_2(x_1, x_3) &= \frac{K}{J_2}(x_1 - x_3), g_2 = \frac{1}{J_2} \end{aligned} \tag{3}$$

III. Direct Adaptive Backstepping Control with Tuning Functions

The approach by tuning functions is developed in [1-2] to avoid the problem of overparameterization and the adaptation laws differentiations as a new form of adaptive backstepping control. The principal advantage is that the number of parameter estimates is minimal, that is equal to the number of unknown parameters [1]. The proposed approach of adaptation is based on the tracking error based parameter adaptation law.

Let us consider the 4^{th} order nonlinear system [8-9] given by

$$\dot{x}_1 = x_2 \tag{4}$$

$$\dot{x}_2 = b_2 x_3 + \varphi_2^T(x_1, x_2)\theta + f_2(x_1, x_2) \tag{5}$$

$$\dot{x}_3 = x_4 \tag{6}$$

$$\dot{x}_4 = b_4 u + \varphi_4^T(x_1, x_2, x_3, x_4)\theta + f_4(x_1, x_2, x_3, x_4) \tag{7}$$

where, the parameter vector $\theta \in \mathbb{R}^p$ is unknown and constant. The nonlinear functions $f_i : \mathbb{R}^i \to \mathbb{R}$ and $\varphi_i : \mathbb{R}^i \to \mathbb{R}^p$ ($i = 1, \cdots, 4$) are known, and the control gains b_i are known. The control objective is to achieve the asymptotic tracking of a reference signal y_r by x_1. The reference signal y_r and its derivatives $\dot{y}_r, \ldots, y_r^{(4)}$ are assumed piecewise continuous and bounded. In the following, we describe the main steps of the controller design for the single-link flexible-joint robot model.

Step 1: We consider the first subsystem (4), the state variable x_2 is treated as a virtual control variable and we define the first desired value $x_{1d} = \alpha_0 = y_r$. The first error is defined by $e_1 = x_1 - \alpha_0$, and its time derivative is given by

$$\dot{e}_1 = \dot{x}_1 - \dot{\alpha}_0 = x_2 - \dot{\alpha}_0 \tag{8}$$

We consider the following Lyapunov function

$$V_1 = \frac{1}{2}e_1^2 \tag{9}$$

The derivative of the Lyapunov function is given by

$$\dot{V}_1 = e_1\dot{e}_1 = e_1\left[x_2 - \dot{\alpha}_0\right] = e_1\left[e_2 + \alpha_1 - \dot{\alpha}_0\right] \tag{10}$$

where $x_2 = e_2 + \alpha_1$. In order to ensure the stability of the first subsystem described by equation (4), we take the desired value of x_2, the function α_1, such as

$$\alpha_1 = x_{2d} = -k_1 e_1 + \dot{\alpha}_0 \tag{11}$$

where $k_1 > 0$. The derivative of the Lyapunov function becomes

$$\dot{V}_1 = -k_1 e_1^2 + e_1 e_2 \tag{12}$$

Step 2: We consider the subsystems (4) and (5), and we define the error $e_2 = x_2 - \alpha_1$, and rewrite the equations of the system in the space (e_1, e_2) as follows

$$\dot{e}_1 = e_2 + \alpha_1 - \dot{\alpha}_0 \tag{13}$$

$$\dot{e}_2 = b_2 x_3 + \varphi_2^T(x_1, x_2)\theta + f_2(x_1, x_2) - \dot{\alpha}_1 \tag{14}$$

We take as the Lyapunov function

$$V_2 = V_1 + \frac{1}{2}e_2^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{15}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ is the estimation error that appears in (5). One computes $\dot{\alpha}_1$ analytically

$$\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial x_1}\dot{x}_1 + \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r + \frac{\partial \alpha_1}{\partial \dot{y}_r}\ddot{y}_r = \frac{\partial \alpha_1}{\partial x_1}x_2 + \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r + \frac{\partial \alpha_1}{\partial \dot{y}_r}\ddot{y}_r \quad (16)$$

The derivative of the Lyapunov function is given by

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + e_2\dot{e}_2 + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} \\ &= -k_1e_1^2 + e_2\Big[e_1 + b_2e_3 + b_2\alpha_2 + \varphi_2^T(x_1,x_2)\hat{\theta} + f_2(x_1,x_2) \\ &\quad - \frac{\partial \alpha_1}{\partial x_1}x_2 - \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r - \frac{\partial \alpha_1}{\partial \dot{y}_r}\ddot{y}_r\Big] + \tilde{\theta}^T\Gamma^{-1}\left(-\dot{\hat{\theta}} + \tau_2\right)\end{aligned} \quad (17)$$

where $x_3 = e_3 + \alpha_2$ and the tuning function τ_2 is defined as

$$\tau_2 = \Gamma\varphi_2(x_1,x_2)e_2 \quad (18)$$

In order to ensure the stability of both subsystems described by equations (4) and (5), we take the desired value of x_3, the function α_2, such as

$$\alpha_2 = \frac{1}{b_2}\Big[-k_2e_2 - e_1 - \varphi_2^T(x_1,x_2)\hat{\theta} - f_2(x_1,x_2) - \frac{\partial \alpha_1}{\partial x_1}x_2 + \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r + \frac{\partial \alpha_1}{\partial \dot{y}_r}\ddot{y}_r\Big] \quad (19)$$

where $k_2 > 0$. The derivative of the Lyapunov function becomes

$$\dot{V}_2 = -k_1e_1^2 - k_2e_2^2 + b_2e_2e_3 + \tilde{\theta}^T\Gamma^{-1}\left(-\dot{\hat{\theta}} + \tau_2\right) \quad (20)$$

Step 3: We consider the subsystems (4), (5) and (6), and we define the error $e_3 = x_3 - \alpha_2$, and rewrite the equations of the system in the space (e_1, e_2, e_3) as follows

$$\dot{e}_1 = e_2 + \alpha_1 - \dot{\alpha}_0 \quad (21)$$

$$\dot{e}_2 = b_2e_3 + b_2\alpha_2 + \varphi_2^T(x_1,x_2)\theta + f_2(x_1,x_2) - \dot{\alpha}_1 \quad (22)$$

$$\dot{e}_3 = x_4 - \dot{\alpha}_2 \quad (23)$$

The Lyapunov function considered here is

$$V_3 = V_2 + \frac{1}{2}e_3^2 \quad (24)$$

One computes $\dot{\alpha}_2$ analytically

$$\begin{aligned}\dot{\alpha}_2 &= \frac{\partial \alpha_2}{\partial x_1}\dot{x}_1 + \frac{\partial \alpha_2}{\partial x_2}\dot{x}_2 + \frac{\partial \alpha_2}{\partial \hat{\theta}}\dot{\hat{\theta}} + \frac{\partial \alpha_2}{\partial y_r}\dot{y}_r + \frac{\partial \alpha_2}{\partial \dot{y}_r}\ddot{y}_r + \frac{\partial \alpha_2}{\partial \ddot{y}_r}\dddot{y}_r \\ &= \frac{\partial \alpha_2}{\partial x_1}x_2 + \frac{\partial \alpha_2}{\partial x_2}b_2x_3 + \frac{\partial \alpha_2}{\partial x_2}\varphi_2^T(x_1,x_2)\theta + \frac{\partial \alpha_2}{\partial x_2}f_2(x_1,x_2) \\ &\quad + \frac{\partial \alpha_2}{\partial \hat{\theta}}\dot{\hat{\theta}} + \frac{\partial \alpha_2}{\partial y_r}\dot{y}_r + \frac{\partial \alpha_2}{\partial \dot{y}_r}\ddot{y}_r + \frac{\partial \alpha_2}{\partial \ddot{y}_r}\dddot{y}_r\end{aligned} \quad (25)$$

The derivative of the Lyapunov function is given by

$$\begin{aligned}\dot{V}_3 &= \dot{V}_2 + e_3\dot{e}_3 \\ &= -k_1e_1^2 - k_2e_2^2 + e_3\Big[b_2e_2 + x_4 - \frac{\partial \alpha_2}{\partial x_1}x_2 - \frac{\partial \alpha_2}{\partial x_2}b_2x_3 \\ &\quad - \frac{\partial \alpha_2}{\partial x_2}\varphi_2^T(x_1,x_2)\hat{\theta} - \frac{\partial \alpha_2}{\partial x_2}f_2(x_1,x_2) - \frac{\partial \alpha_2}{\partial \hat{\theta}}\dot{\hat{\theta}} - \frac{\partial \alpha_2}{\partial y_r}\dot{y}_r \\ &\quad - \frac{\partial \alpha_2}{\partial \dot{y}_r}\ddot{y}_r - \frac{\partial \alpha_2}{\partial \ddot{y}_r}\dddot{y}_r\Big] + \tilde{\theta}^T\Gamma^{-1}\left(-\dot{\hat{\theta}} + \tau_3\right)\end{aligned} \quad (26)$$

where $x_4 = e_4 + \alpha_3$ and the tuning function τ_3 is defined as

$$\tau_3 = \tau_2 - \Gamma\varphi_2(x_1,x_2)\frac{\partial \alpha_2}{\partial x_2}e_3 \quad (27)$$

In order to ensure the stability of both subsystems described by equations (4), (5) and (6), we take the desired value of x_4, the function α_3, such as

$$\begin{aligned}\alpha_3 &= -k_3e_3 - b_2e_2 + \frac{\partial \alpha_2}{\partial x_1}x_2 + \frac{\partial \alpha_2}{\partial x_2}b_2x_3 + \frac{\partial \alpha_2}{\partial x_2}\varphi_2^T(x_1,x_2)\hat{\theta} \\ &\quad + \frac{\partial \alpha_2}{\partial x_2}f_2(x_1,x_2) + \frac{\partial \alpha_2}{\partial \hat{\theta}}\tau_3 + \frac{\partial \alpha_2}{\partial y_r}\dot{y}_r + \frac{\partial \alpha_2}{\partial \dot{y}_r}\ddot{y}_r + \frac{\partial \alpha_2}{\partial \ddot{y}_r}\dddot{y}_r\end{aligned} \quad (28)$$

where $k_3 > 0$. The derivative of the Lyapunov function is reduced to

$$\begin{aligned}\dot{V}_3 &= -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 + e_3e_4 + e_3\frac{\partial \alpha_2}{\partial \hat{\theta}}\left(-\dot{\hat{\theta}} + \tau_3\right) \\ &\quad + \tilde{\theta}^T\Gamma^{-1}\left(-\dot{\hat{\theta}} + \tau_3\right)\end{aligned} \quad (29)$$

Step 4: We consider the subsystems (4), (5), (6) and (7), and we introduce the error $e_4 = x_4 - \alpha_3$, and rewrite the system equations in the space (e_1, e_2, e_3, e_4) as follows

$$\dot{e}_1 = e_2 + \alpha_1 - \dot{\alpha}_0 \quad (30)$$

$$\dot{e}_2 = b_2e_3 + b_2\alpha_2 + \varphi_2^T(x_1,x_2)\theta + f_2(x_1,x_2) - \dot{\alpha}_1 \quad (31)$$

$$\dot{e}_3 = e_4 + \alpha_3 - \dot{\alpha}_2 \quad (32)$$

$$\dot{e}_4 = b_4u + \varphi_4^T(x_1,x_2,x_3,x_4)\theta + f_4(x_1,x_2,x_3,x_4) - \dot{\alpha}_3 \quad (33)$$

We take as the Lyapunov function

$$V_4 = V_3 + \frac{1}{2}e_4^2 \tag{34}$$

One computes $\dot{\alpha}_3$ analytically

$$\begin{aligned}\dot{\alpha}_3 &= \frac{\partial\alpha_3}{\partial x_1}\dot{x}_1 + \frac{\partial\alpha_3}{\partial x_2}\dot{x}_2 + \frac{\partial\alpha_3}{\partial x_3}\dot{x}_3 + \frac{\partial\alpha_3}{\partial\hat{\theta}}\dot{\hat{\theta}} + \frac{\partial\alpha_3}{\partial y_r}\dot{y}_r + \frac{\partial\alpha_3}{\partial\dot{y}_r}\ddot{y}_r + \frac{\partial\alpha_3}{\partial\ddot{y}_r}\dddot{y}_r + \frac{\partial\alpha_3}{\partial\dddot{y}_r}\ddddot{y}_r \\ &= \frac{\partial\alpha_3}{\partial x_1}x_2 + \frac{\partial\alpha_3}{\partial x_2}b_2x_3 + \frac{\partial\alpha_3}{\partial x_2}\varphi_2^T(x_1,x_2)\theta + \frac{\partial\alpha_3}{\partial x_2}f_2(x_1,x_2) + \frac{\partial\alpha_3}{\partial x_3}x_4 + \frac{\partial\alpha_3}{\partial\hat{\theta}}\dot{\hat{\theta}} \\ &+ \frac{\partial\alpha_3}{\partial y_r}\dot{y}_r + \frac{\partial\alpha_3}{\partial\dot{y}_r}\ddot{y}_r + \frac{\partial\alpha_3}{\partial\ddot{y}_r}\dddot{y}_r + \frac{\partial\alpha_3}{\partial\dddot{y}_r}\ddddot{y}_r\end{aligned} \tag{35}$$

The derivative of the Lyapunov function is given by

$$\begin{aligned}\dot{V}_4 &= \dot{V}_3 + e_4\dot{e}_4 \\ &= -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 + e_4\Big[b_4u + \varphi_4^T(x_1,x_2,x_3,x_4)\hat{\theta} \\ &+ f_4(x_1,x_2,x_3,x_4) - \frac{\partial\alpha_3}{\partial x_1}x_2 - \frac{\partial\alpha_3}{\partial x_2}b_2x_3 - \frac{\partial\alpha_3}{\partial x_2}\varphi_2^T(x_1,x_2)\hat{\theta} \\ &- \frac{\partial\alpha_3}{\partial x_2}f_2(x_1,x_2) - \frac{\partial\alpha_3}{\partial x_3}x_4 - \frac{\partial\alpha_3}{\partial\hat{\theta}}\dot{\hat{\theta}} - \frac{\partial\alpha_3}{\partial y_r}\dot{y}_r - \frac{\partial\alpha_3}{\partial\dot{y}_r}\ddot{y}_r \\ &- \frac{\partial\alpha_3}{\partial\ddot{y}_r}\dddot{y}_r - \frac{\partial\alpha_3}{\partial\dddot{y}_r}\ddddot{y}_r\Big] + e_3\frac{\partial\alpha_2}{\partial\hat{\theta}}\left(-\dot{\hat{\theta}} + \tau_3\right) + \tilde{\theta}^T\Gamma^{-1}\left(-\dot{\hat{\theta}} + \tau_4\right)\end{aligned} \tag{36}$$

Then, we take the control law as

$$\begin{aligned}u &= \frac{1}{b_4}\Big[-k_4e_4 - \varphi_4^T(x_1,x_2,x_3,x_4)\hat{\theta} - f_4(x_1,x_2,x_3,x_4) \\ &+ \frac{\partial\alpha_3}{\partial x_1}x_2 + \frac{\partial\alpha_3}{\partial x_2}b_2x_3 + \frac{\partial\alpha_3}{\partial x_2}\varphi_2^T(x_1,x_2)\hat{\theta} + \frac{\partial\alpha_3}{\partial x_2}f_2(x_1,x_2) \\ &+ \frac{\partial\alpha_3}{\partial x_3}x_4 + \frac{\partial\alpha_3}{\partial\hat{\theta}}\tau_4 + \frac{\partial\alpha_3}{\partial y_r}\dot{y}_r + \frac{\partial\alpha_3}{\partial\dot{y}_r}\ddot{y}_r + \frac{\partial\alpha_3}{\partial\ddot{y}_r}\dddot{y}_r + \frac{\partial\alpha_3}{\partial\dddot{y}_r}\ddddot{y}_r \\ &- e_3\frac{\partial\alpha_2}{\partial\hat{\theta}}\Gamma\left(\varphi_2(x_1,x_2)\frac{\partial\alpha_3}{\partial x_2} - \varphi_4(x_1,x_2,x_3,x_4)\right)\Big]\end{aligned} \tag{37}$$

Substituting the control law into $\dot{V}_4$ gives

$$\begin{aligned}\dot{V}_4 &= -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 + e_4\Bigg[\frac{\partial\alpha_3}{\partial\hat{\theta}}\left(-\dot{\hat{\theta}} + \tau_4\right) \\ &- e_3\frac{\partial\alpha_2}{\partial\hat{\theta}}\Gamma\left(\varphi_2(x_1,x_2)\frac{\partial\alpha_3}{\partial x_2} - \varphi_4(x_1,x_2,x_3,x_4)\right)\Bigg] \\ &+ e_3\frac{\partial\alpha_2}{\partial\hat{\theta}}\left(-\dot{\hat{\theta}} + \tau_3\right) + \tilde{\theta}^T\Gamma^{-1}\left(-\dot{\hat{\theta}} + \tau_4\right) \\ &= -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 + e_4\frac{\partial\alpha_3}{\partial\hat{\theta}}\left(-\dot{\hat{\theta}} + \tau_4\right) \\ &+ e_3\frac{\partial\alpha_2}{\partial\hat{\theta}}\left(-\dot{\hat{\theta}} + \tau_4\right) + \tilde{\theta}^T\Gamma^{-1}\left(-\dot{\hat{\theta}} + \tau_4\right)\end{aligned} \tag{38}$$

The new tuning function τ_4 is defined as

$$\tau_4 = \tau_3 - \Gamma\left(\varphi_2(x_1,x_2)\frac{\partial\alpha_3}{\partial x_2} - \varphi_4(x_1,x_2,x_3,x_4)\right)e_4 \tag{39}$$

where $k_4 > 0$. The update law is selected as

$$\begin{aligned}\dot{\hat{\theta}} = \tau_4 = \Gamma\Bigg(&\varphi_2(x_1,x_2)e_2 - \varphi_2(x_1,x_2)\frac{\partial\alpha_2}{\partial x_2}e_3 \\ &-\varphi_2(x_1,x_2)\frac{\partial\alpha_3}{\partial x_2}e_4 + \varphi_4(x_1,x_2,x_3,x_4)e_4\Bigg)\end{aligned} \tag{40}$$

with the choice: $\dot{\hat{\theta}} = \tau_4$, the derivative of the Lyapunov function is reduced to

$$\dot{V}_4 = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 \le 0 \tag{41}$$

This directly leads to the asymptotic stability of the errors of the closed-loop system. Barbalat's lemma [10] can be used to show that $\dot{V} \to 0$ as $t \to \infty$. Thus e_1, e_2, e_3 and $e_4 \to 0$ as $t \to \infty$.

IV. Simulation Results

The single-link flexible-joint robot model used in this paper is given by (2) where the parameter values are given in Table. 1 [6].

Table 1. Single-link flexible-joint robot model parameters.

Symbol	*Value*	*Unit*
g	9,81	[m/s^2]
M	1	[*kg*]
L	1	[*m*]
J_1	0.4	[$kg.m^2$]
J_2	0.02	[$kg.m^2$]
K	100	[*N.m/rad*]

For simulation, the unknown parameter θ of the system is selected as $\theta = MgL$. Our objective is to force the output of the system to follow the reference trajectory given by: $y_d = 0.1\sin(t)$. The tuning functions τ_2, τ_3 and τ_4 are defined as

$$\tau_2 = -\frac{\Gamma}{J_1}\sin(x_1)e_2 \tag{42}$$

$$\tau_3 = \tau_2 + \frac{\Gamma}{J_1}\sin(x_1)\frac{\partial\alpha_2}{\partial x_2}e_3 \tag{43}$$

$$\tau_4 = \tau_3 + \frac{\Gamma}{J_1}\sin(x_1)\frac{\partial\alpha_3}{\partial x_2}e_4 \tag{44}$$

where $\Gamma > 0$. The stabilizing functions α_1, α_2 and α_3 are given by

$$\alpha_1 = x_{2d} = -k_1 e_1 + \dot{\alpha}_0 \qquad (45)$$

$$\alpha_2 = x_{3d} = \frac{1}{g_1}\left[-k_2 e_2 - e_1 + \frac{\hat{\theta}}{J_1}\sin(x_1) + \frac{K}{J_1}x_1 + \frac{\partial \alpha_1}{\partial x_1}x_2 + \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r + \frac{\partial \alpha_1}{\partial \dot{y}_r}\ddot{y}_r\right] \qquad (46)$$

$$\alpha_3 = x_{4d} = -k_3 e_3 - g_1 e_2 + \frac{\partial \alpha_2}{\partial x_1}x_2 - \frac{\partial \alpha_2}{\partial x_2}\frac{\hat{\theta}}{J_1}\sin(x_1) - \frac{\partial \alpha_2}{\partial x_2}\frac{K}{J_1}x_1 + \frac{\partial \alpha_2}{\partial x_2}g_1 x_3 + \frac{\partial \alpha_2}{\partial \hat{\theta}}\tau_3 + \frac{\partial \alpha_2}{\partial y_r}\dot{y}_r + \frac{\partial \alpha_2}{\partial \dot{y}_r}\ddot{y}_r + \frac{\partial \alpha_2}{\partial \ddot{y}_r}\dddot{y}_r \qquad (47)$$

The control law u is given by

$$u = \frac{1}{g_2}\left[-k_4 e_4 - e_3 - \frac{K}{J_2}(x_1 - x_3) + \frac{\partial \alpha_3}{\partial x_1}x_2 + \frac{\partial \alpha_3}{\partial x_3}x_4 - \frac{\partial \alpha_3}{\partial x_2}\frac{\hat{\theta}}{J_1}\sin(x_1) - \frac{\partial \alpha_3}{\partial x_2}\frac{K}{J_1}x_1 + \frac{\partial \alpha_3}{\partial x_2}g_1 x_3 + \frac{\partial \alpha_3}{\partial \hat{\theta}}\tau_4 + \frac{\partial \alpha_3}{\partial y_r}\dot{y}_r + \frac{\partial \alpha_3}{\partial \dot{y}_r}\ddot{y}_r + \frac{\partial \alpha_3}{\partial \ddot{y}_r}\dddot{y}_r + \frac{\partial \alpha_3}{\partial \dddot{y}_r}\ddddot{y}_r - e_3\frac{\partial \alpha_2}{\partial \hat{\theta}}\frac{\Gamma}{J_1}\sin(x_1)\frac{\partial \alpha_3}{\partial x_2}\right] \qquad (48)$$

The update law is given by

$$\dot{\hat{\theta}} = \tau_4 = \frac{\Gamma}{J_1}\sin(x_1)\left(-e_2 + \frac{\partial \alpha_2}{\partial x_2}e_3 + \frac{\partial \alpha_3}{\partial x_2}e_4\right) \qquad (49)$$

where $\Gamma > 0$.

The selected initial conditions are: $\hat{\theta}(0) = 0$ and $x(0) = \left[0.1 \quad 0 \quad 0.1 + \frac{MgL}{K}\sin(0.1) \quad 0\right]^T$. The design parameters are selected as follows: $k_1 = 1$, $k_2 = 5$, $k_3 = 7500$, $k_4 = 5$ and $\Gamma = 0.015$.

Simulation results are shown in figures 2 to 11. Figures 2 to 5 show actual and desired trajectories of the angular position and velocity of the link and the motor shaft. Figures 6 to 9 show the trajectories of the tracking errors. Figure 10 shows the trajectory of the control input signal u. Figure 11 shows the trajectory of the estimated parameter $\hat{\theta}$.

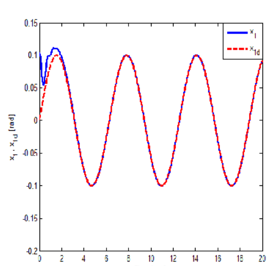

Fig. 2. Angular position of the link: actual x_1("-") and desired x_{1d} ("--").

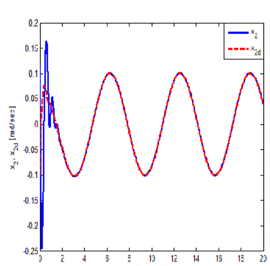

Fig. 3. Angular velocity of the link: actual x_2 ("-") and desired x_{2d} ("--").

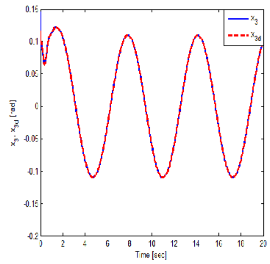

Fig. 4. Angular position of the motor shaft: actual x_3 ("-") and desired x_{3d} ("--").

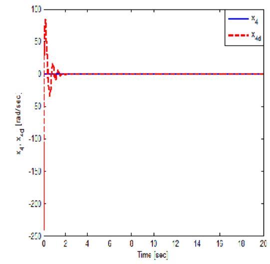

Fig. 5. Angular velocity of the motor shaft: actual x_4 ("-") and desired x_{4d} ("--").

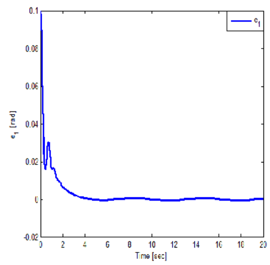

Fig. 6. Tracking error e_1.

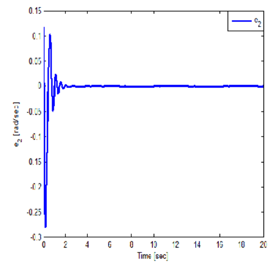

Fig. 7. Tracking error e_2.

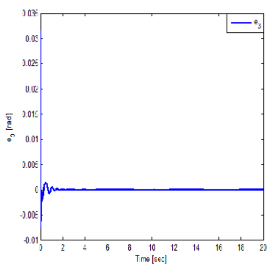

Fig. 8. Tracking error e_3.

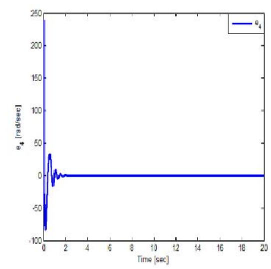

Fig. 9. Tracking error e_4.

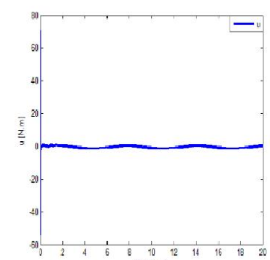

Fig. 10. Control input u.

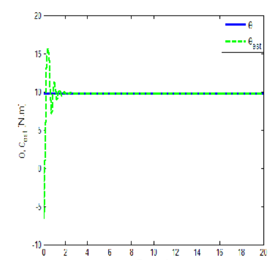

Fig. 11. Estimated parameter $\hat{\theta}$.

We can see that the control performances, the tracking and the parameter estimation of the single-link flexible-joint robot system using direct adaptive backstepping control with tuning functions are effective.

V. Conclusions

In this paper, the adaptive backstepping control with tuning functions approach for a single-link flexible-joint robot model is proposed to estimate the parameters. The control law and the parameter update law are designed along with the Lyapunov function to guarantee global stability. The simulation results are given to illustrate that the proposed approach is effective and gives good tracking, good parameter estimation and good control tracking performances.

REFERENCES

[1] M. Krstic, I. Kanellakopoulos and P. V. Kokotovic, "Adaptive nonlinear control without overparametrization," *Systems and Control Letters, North-Holland,* Vol. 19, No. 3, pp. 177-185, 1992.

[2] M. Krstic, I. Kanellakopoulos and P. V. Kokotovic, *"Nonlinear and adaptive control design,"* Wiley-Interscience Publication, New York, 1995.

[3] I. Kanellakopoulos, P. V. Kokotovic, and A. S. Morse, "Systematic design of adaptive controllers for feedback linearizable systems," *IEEE Trans Autom Control,* Vol. 36, No. 11, pp. 1241-1253, 1991.

[4] Y. Li, S. Tong and T. Li, ''Fuzzy adaptive dynamic surface control for a single link flexible joint robot," *Nonlinear Dyn,* Vol. 70, No. 3, pp. 2035-2048, 2012.

[5] A. R. Sahab and M. R. Modabbernia, ''Backstepping method for a single link flexible joint manipulator using genetic algorithm," *International Journal of Innovative Computing Information and Control,* Vol. 7, No. 7, pp. 4161-4170, 2011.

[6] S. Nicosia and P. Tomei, ''A tracking controller flexible joint robots using only link position feedback," *IEEE Transactions on Automatic Control,* Vol. 40, No. 5, pp. 885-890, 1995.

[7] F. Wang and X. Liu, ''Fuzzy H-inf control of flexible joint robot," *ICICA, Springer-Verlag Berlin Heidelberg,* pp. 424-431, 2011.

[8] Y. Soukkou and S. Labiod, ''Adaptive backstepping control using combined direct and indirect adaptation based gradient update law for a single-link flexible-joint robot," *5th International Conference on Electrical Engineering and Automatic Control, Setif, Algeria,* November 24-26, 2013.

[9] Y. Soukkou and S. Labiod, ''Adaptive backstepping control using combined direct and indirect adaptation based least squares update law for a single-link flexible-joint robot," *Proceedings of the 3rd International Conference on Industrial Engineering and Manufacturing, Batna, Algeria,* pp. 272-279, May 11-13, 2014.

[10] H. K. Khalil, "Nonlinear systems," Prentice Hall, Englewood Cliffs, New Jersey, 1996.